THE EFFECT OF CURVATURE OF THE EQUILIBRIUM LINE ON THE FILM RESISTANCES IN GAS ABSORPTION

by

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INTRODUCTION

A convenient method for estimating the height of a countercurrent absorption tower is based on the concept of the height of a transfer unit, which was introduced by Chilton and Colburn (1,2). In practical design work this concept has the following advantages: 1) the procedure is simple, direct, and rapid, especially for rough estimates; 2) the mass-transfer resistance of the packing is expressed in terms of an experimentally determined number which has the dimensions of length only; 3) the procedure closely parallels that used in the design of plate towers.

The basic concept is that the estimation of the height of a tower always depends upon the evaluation of a definite integral. The value of this integral, which is dimensionless, expresses the difficulty of absorbing the solute from the gas; it is greater when the mean driving force is smaller and when the required change in gas composition is larger. Chilton and Colburn (1,2) have called it the "number of transfer units" --- N.T.U.

Approximate analytical solutions for evaluation of N.T.U. have been proposed by Colburn (1,2), Onda and Sada (3), Ramm (4), Scheibel and Othmer (5,6), Wiegand (7) and White (8). Graphical step methods were proposed by Baker (9), White (10) and Ogawa (11), but the standard method is by graphical integration.

In any gas-liquid diffusion process, the materials are often considered to diffuse across two films, the gas-film and the liquid-film. In order to follow the effect of various changing factors, the resistances of the gas-film and the liquid-film must be considered separately. According to the two-film theory, which was proposed by Whitman (12), the total resistance to diffusion is made up of the gas-film and the liquid-film in series. At the interface it is supposed that the two films are in equilibrium. At the rates of mass transfer used in gas absorption this is probably a very good assumption (12).

THEORETICAL CONSIDERATIONS

Review of Determination of Number of Transfer Units

Consider the evaluation of the number of transfer units required for gas absorption, Onda and Sada (3) using mole ratio or weight ratio of solute units derived the following formula.

$$N_{toY} = \int_{Y_a}^{Y_b} \frac{dY}{\ln\left(\frac{1 + Y}{1 + Y^*}\right)} \qquad (1)$$

$$N_{tY} = \int_{Y_a}^{Y_b} \frac{dY}{\ln\left(\frac{1 + Y}{1 + Y_i}\right)} \qquad (2)$$

Several methods are available for the solution of the above Equations (1) or (2).

Graphical Integration. A rigorous solution of Equation (1) is possible using the necessary data of Y and Y*. Y corresponds to the values on the operating line and Y* to those of the equilibrium curve on the same coordinate X. The number of transfer units can be evaluated graphically by determining the areas under a curve of $\frac{1}{\ln(\frac{1 + Y}{1 + Y^*})}$ as ordinate against Y as abscissa. The subscript a corresponds to the top of the tower and b corresponds to the bottom of the tower. In Equation (2) a tie-line slope of $(- \frac{k_X}{k_Y})$ is assumed and this can be plotted. The tie-line intersects the equilibrium curve at Y_i. The number of transfer units can be evaluated graphically by using Y_i instead of Y^*. Although, a good answer may be obtained by using the technique of graphical integration, it is a long time-consumming process to calculate the value of N_{toY} or N_{tY}.

Simplification of the Integral. For diffusion of solute through a stationary component as in gas absorption, Wiegand (6) showed that the logarithmic mean value may be approximated by the arithmetic average of $\frac{1}{1 + Y}$ and $\frac{1}{1 + Y^*}$. If the ratio of these two quantities is less than 1.4 the error is less than 1 %. If the small error introduced by this approximation is acceptable, Equations (1) and (2) become equal to :

$$N_{toY} = \int_{Y_a}^{Y_b} \frac{dY}{(Y - Y^*)} - \frac{1}{2} \ln \frac{1 + Y_2}{1 + Y_1} \qquad (3)$$

$$N_{tY} = \int_{Y_a}^{Y_b} \frac{dY}{(Y - Y_i)} - \frac{1}{2} \ln \frac{1 + Y_2}{1 + Y_1} \qquad (4)$$

Normally the integral term must be evaluated graphically, but it is much simpler than the integration of Equation (1) or (2). In many instances, the last terms of Equations (3) and (4) are negligible. For dilute solutions $1 + Y$ and $1 + Y^*$ are approximately equal to 1. In this case $\ln(\frac{1 + Y}{1 + Y^*})$ equals $Y - Y^*$ which leads to the same result. Hence graphical evaluation of the integral term is frequently satisfactory.

Stepwise Methods. A simple graphical procedure for evaluating the number of transfer units needed to accomplish a certain change has been proposed by Baker (9). Baker based his proposal upon the use of an arithmetic average of the driving forces and the assumption that, over the length of one transfer unit, both operating line and equilibrium curve are essentially linear. The procedure is to draw an intermediate line between the operating line and the equilibrium curve in such a way that it divides the vertical distance along the tie-line into equal parts. A line is drawn horizontally from the terminal point on the operating line, till it meets the middle line and then extended an equal distance beyond. From this point, a line is then drawn vertically until it reaches the operating line. One step like this is equivalent to one transfer unit. The procedure is continued till the terminal composition, at the bottom of the tower, is reached. When the liquid-film resistance controls, the graphical procedure should be modified by locating a line half way horizontally between the operating line and the equilibrium curve. The step should be taken vertically away from the operating line rather than horizontally. Thus a theoretical plate step is more effective than a transfer unit when mV'/L' is less than unity; the reverse is true if mV'/L' is greater than unity.

A second graphical method was suggested by White (10). This method allows intermediate cases to be handled and the resistance is divided between the films. The procedure in White's method will be described later in this report.

A third graphical method suggested by Ogawa (11) involves a technique of stepping off fractional units. The Ogawa method is only a modification of White's method for graphical calculation of transfer units in countercurrent operations.

Rapid Evaluation of Transfer Units. Approximations of the integration of various equations which describe the number of transfer units are given by Colburn (1,2), for use in operating conditions where both the operating line and the equilibrium curve are nearly linear. Slightly different results applicable to such cases were obtained by Othmer and Scheibel (6).

White (8) formulated a method in which he assumed that the portion of the equilibrium curve beneath the operating line may be represented by a parabola. The mean driving force may be calculated from the terminal driving forces and the central driving force by means of a nomograph.

Onda and Sada (3) have suggested approximate formulas for use when the number of transfer units is large. They also derived formulas for use when the equilibrium curve is straight or curved.

White's Method of Graphical Stepwise Construction of Transfer Units

White's method of graphical stepwise construction has two advantages over the method of Baker. It can be used for stepping off transfer units in either direction, while Baker's method must be applied in one direction only. White's method includes all tie-line slopes between liquid-film and gas-film resistances controlling, while Baker's method is limited to the cases of gas-film resistance or liquid-film resistance controlling. Ogawa's method is a modification of White's method which permits steps corresponding to fractional transfer units.

Because White's method provides considerable insight into the behavior of gas absorption tower operation and the significance of transfer units, this method will be treated in considerable detail. To demonstrate White's method (10), an approximate graphical stepwise construction for transfer units is shown in diagrams in order to clarify the difference between a transfer unit and a theoretical plate.

The assumptions made by Chilton and Colburn (1,2) concerning a countercurrent tower are as follows: 1) the mean partial pressure of the inert carrier gas remains constant and equal to the total pressure throughout the tower; 2) the transfer coefficients k_Xa and k_Ya remain constant.

The solute material balance follows,

$$V'dY = L'dX \tag{5}$$

This balance is expressed in the following units.

$$\left(\frac{\text{lb inert gas}}{\text{hr}}\right)\left(\frac{\text{lb solute}}{\text{lb inert gas}}\right)=\left(\frac{\text{lb inert liquid}}{\text{hr}}\right)\left(\frac{\text{lb solute}}{\text{lb inert liquid}}\right)$$

Rearranging Equation (5),

$$\frac{dY}{dX} = \frac{L'}{V'} = \text{a constant.} \tag{6}$$

The slope is constant because L' and V' are constant and the ratio of two constants is also constant. A line whose slope is constant is straight.

Case 1. Figure 1 shows the stepwise construction of N.T.U. when the gas-film is the controlling factor. The distance along the operating line AD corresponding to one transfer unit is defined approximately as:

$$N_{toY} = \frac{\Delta Y}{Y - Y^*} = 1. \tag{7}$$

Note that AE = EF, BE = EH, and CF = BH. But CF is ΔY, while BH = $(Y - Y^*)$ at the midpoint of the transfer unit step. This distance BH can be taken as a close approximation of the average value of $(Y - Y^*)$, unless the curvature of the equilibrium curve is excessive. The geometry involved is simplified by drawing the lines AH and HC. The first of these has the slope s_Y and the second the slope t_Y.

$$s_Y = - \frac{L'}{V'} \tag{8}$$

$$t_Y = 3 \frac{L'}{V'} \tag{9}$$

One transfer unit is the distance AC. The succeeding steps are determined by repeating the procedure from C toward D. The remaining length of the operating line over the distance along the line extended to the end of the step may be considered a fractional step.

Case 2. Figure 2 shows the stepwise construction when the liquid-film is the controlling factor.

$$N_{toX} = \frac{\Delta X}{X^* - X} = 1. \tag{10}$$

Note that CE = EF, BE = EH, AF = BH, AF = ΔX, and BH = $X^* - X$.

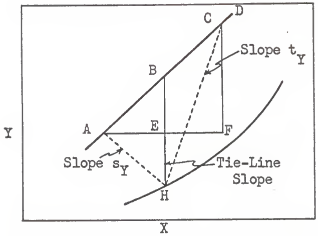

Fig. 1. Graphical stepwise construction of a transfer unit for the case of negligible liquid film resistance in gas absorption.

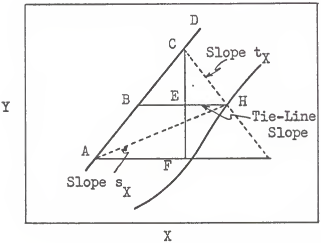

Fig. 2. Graphical stepwise construction of a transfer unit for the case of negligible gas film resistance in gas absorption.

Hence AC represents one transfer unit. The slope of AH is s_X and the slope of CH is t_X.

$$s_X = \frac{1}{3}\frac{L'}{V'} \tag{11}$$

$$t_X = -\frac{L'}{V'} \tag{12}$$

Case 3. Figure 3 shows the stepwise construction when both films are important but the gas-film resistance is the more important. The tie-line is here shows as line BK. Note that AE = EF, BE = EH, CF = BH, CF = Δ Y, and BH = $Y - Y_i$ are the same as before, or in Equations (8) and (9), but the slopes of the dotted lines differ.

$$s_Y = -\frac{1}{\frac{V'}{L'} + 2\frac{k_Y}{k_X}} \tag{13}$$

$$t_Y = \frac{3}{\frac{V'}{L'} - 2\frac{k_Y}{k_X}} \tag{14}$$

Case 4. Figure 4 shows the stepwise construction when the resistances of both films are important, but the liquid-film resistance is the more important. BK is the tie-line. Note that CE = EF, BE = EH, AF = BH, AF = Δ X and BH = $X_i - X$. The slopes of the dotted lines are indicated in Equations (15) and (16).

$$s_X = \frac{1}{3}\left(\frac{L'}{V'} - 2\frac{k_X}{k_Y} \right) \tag{15}$$

$$t_X = -\left(\frac{L'}{V'} + 2\frac{k_X}{k_Y} \right) \tag{16}$$

Equations (8) and (9) are special cases of (13) and (14), when the tie-line slope equals infinity; and similarly Equations (11) and (12) are special cases of Equations (15) and (16) when the tie-line slope equals zero.

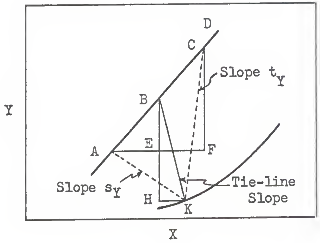

Fig. 3. Graphical stepwise construction of a transfer unit for the case of gas film resistance controlling in gas absorption.

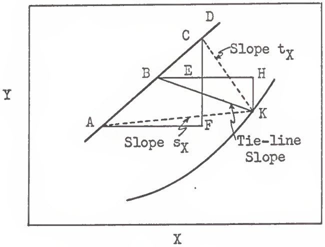

Fig. 4. Graphical stepwise construction of a transfer unit for the case of liquid film resistance controlling in gas absorption.

Derivation of the Conditions for Changing the Construction of a Transfer Unit from Gas Film Controlling to Liquid Film Controlling

Experience with the last two cases has shown that they do not lead to the same number of transfer units, when applied to the same problem. When the gas-film resistance is most important case 3 --- Figure 3 --- is correct. When the liquid-film resistance become most important case 3 gives far too many transfer units. Therefore, it was necessary to determine the point at which a change to case 4 must be made.

Substitute into Equations (13), (14), (15), and (16) the following, $\frac{k_X}{k_Y} = a$, $\frac{L'}{V'} = c$.

$$s_Y = -\frac{ac}{2c + a} \qquad (13')$$

$$t_Y = \frac{3ac}{a - 2c} \qquad (14')$$

$$s_X = \frac{c - 2a}{3} \qquad (15')$$

$$t_X = -(c + 2a) \qquad (16')$$

For the purpose of determining the intersections, let a = c. This leads to the following results.

$$s_Y = -\frac{c}{3}, \quad t_Y = -3c, \quad s_X = -\frac{c}{3}, \quad t_X = -3c.$$

Note that $t_Y \longrightarrow \infty$ as $a \longrightarrow 2c$, and also that $s_X \longrightarrow 0$ as $a \longrightarrow \frac{c}{2}$.

Table 1 illustrates the effect of various tie-line slopes on the slopes of the stepwise construction lines s_Y, t_Y, s_X and t_X for the cases in which c, the slope of the operating line, equals 1.0.

Figure 5 shows the results given in Table 1. The intersections always occur when a = c. Therefore, the change from

Table 1. The Effect of the Tie-Line Slope on the Slopes of the Construction Lines s_Y, t_Y, s_X, and t_X for the Case of c = 1.0.

a	$s_Y = -\frac{a}{a+2}$	$t_Y = \frac{3a}{a-2}$	$s_X = \frac{1}{3}(1-2a)$	$t_X = -(1+2a)$
0.1	-0.048	-0.158	+0.267	-1.20
0.2	-0.091	-0.333	+0.200	-1.40
0.5	-0.200	-1.000	+0.000	-2.00
1.0	-0.333	-3.000	-0.333	-3.00
2.0	-0.500	$-\infty$	-1.000	-5.00
5.0	-0.714	+5.000	-3.000	-11.00
10.0	-0.833	+3.750	-6.333	-21.00
20.0	-0.909	+3.333	-13.000	-41.00
50.0	-0.961	+3.125	-33.000	-101.00
100.0	-0.980	+3.061	-66.333	-201.00

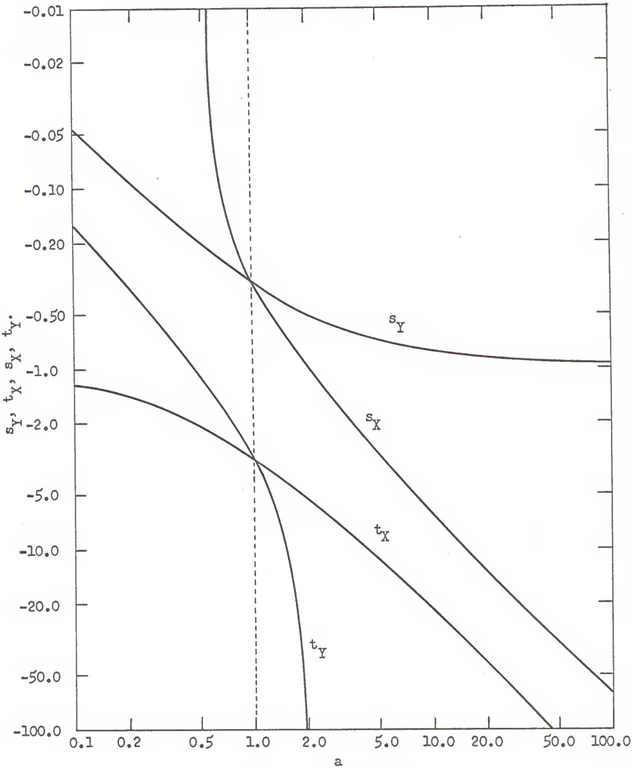

Fig. 5. The effect of the tie-line slope on the slopes of the construction lines s_Y, t_Y, s_X, and t_X for the case of $c = 1.0$.

case 3 to case 4 should be made at the point where the tie-line slope becomes equal to minus the operating line slope.

PRESENTATION OF RESULTS

An Equilibrium Curve That Is S-Shaped Sulfuric Acid-Water-Air System

Calculations of Number of Transfer Units. In order to investigate the effect of curvature of the equilibrium curve, first let us choose the system sulfuric acid-water-air. The equilibrium curve of this system is S-shaped. The equilibrium data for this system were calculated from the vapor pressures given by Greenewalt (13) and Landolt-Börnstein (14) as shown in Table 2. In Table 2 it is assumed that the operation is isothermal at 25°C. and that the mass transfer takes place without chemical reaction.

Since the enthalpy of solution of water in sulfuric acid is considerable it is desirable to investigate adiabatic conditions together with the temperatures reached are given in Table 3. This calculation was based on the following terminal conditions. Moist air is to be dried from a humidity 0.010 pound water per pound dry air to 0.003 pound water per pound dry air by countercurrent scrubbing with 66.6 per cent sulfuric acid, liquid and gas both entering at 25°C. The sensible heat change for the gas was negligible. The necessary data were taken from Hougen et. al. (15).

Figure 6 shows the equilibrium curve for this system under isothermal and adiabatic conditions.

Table 2. Calculation of X vs. Y About the System of Sulfuric Acid-Water-Air at 25°C. Isothermal Operation.

Greenewalt (13)				Landolt-Börnstein (14)			
Vapor pressure mm. Hg	Y $\frac{\text{lb } H_2O}{\text{lb dry air}}$	H_2SO_4 wt%	X $\frac{\text{lb } H_2O}{\text{lb } H_2SO_4}$	Vapor pressure mm. Hg	Y $\frac{\text{lb } H_2O}{\text{lb dry air}}$	H_2SO_4 wt%	X $\frac{\text{lb } H_2O}{\text{lb } H_2SO_4}$
22.4	0.0188	10	9.00				
20.8	0.0175	20	4.00				
19.4	0.0163	25	3.00	19.516	0.01638	24.26	3.120
17.8	0.0149	30	2.34	16.610	0.01390	33.10	1.720
15.8	0.0132	35	1.86	14.613	0.01215	37.69	1.651
13.5	0.0112	40	1.50				
10.9	0.0090	45	1.22	11.557	0.00959	43.75	1.290
8.45	0.0070	50	1.00	7.892	0.00651	52.13	0.915
6.15	0.0051	55	0.82	5.135	0.00422	57.65	0.734
3.97	0.0033	60	0.67	3.024	0.00248	64.47	0.551
2.24	0.0018	65	0.54				
1.03	0.0008	70	0.43	1.125	0.00092	73.13	0.367
0.408	0.0003	75	0.33				
0.124	0.0001	80	0.25				
0.039	0.0000	85	0.18	0.184	0.00015	84.48	0.184

Table 3. Results of the Calculation of the Equilibrium Curve in the System Sulfuric Acid-Water-Air. Adiabatic Condition. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 pound of water per pound of H_2SO_4 (66.6%). Liquid and gas both enter at 25°C. Neglect the sensible-heat change for the gas.

Temperature °C.	X $\frac{\text{lb } H_2O}{\text{lb } H_2SO_4}$	Y $\frac{\text{lb } H_2O}{\text{lb dry air}}$
25.0	0.42	0.0009
59.5	0.60	0.0032
77.4	0.65	0.0050
110.0	0.70	0.0074
115.0	0.75	0.0100

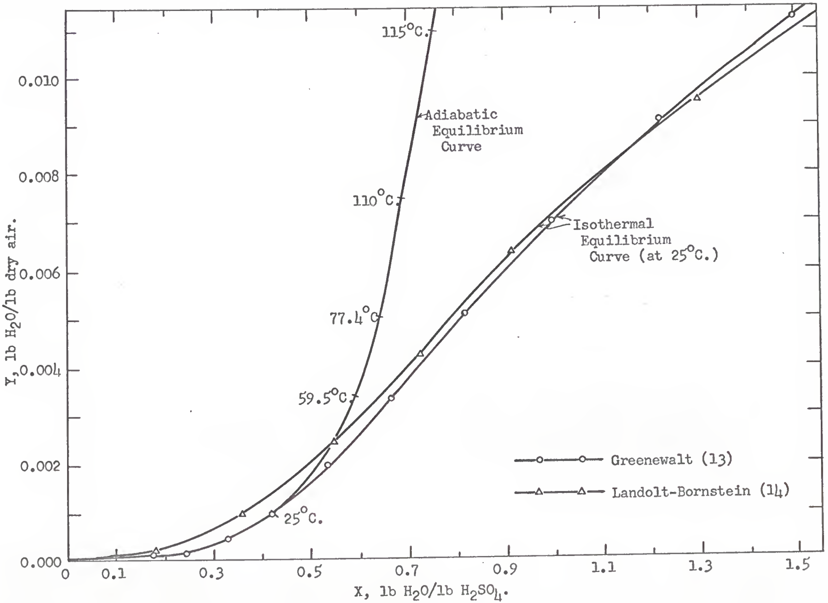

Fig. 6. The adiabatic and isothermal equilibrium curves of sulfuric acid-water-air system.

The results of the graphical stepwise constructions for the cases in which the gas-film is more important than the liquid-film are shown in Table 4. One of the graphical stepwise constructions for the case of a tie-line slope of minus 0.01 is shown in Figure 7. Also shown in Table 4 are the results of two formal graphical integrations.

The case in which the liquid-film is more important than the gas-film is shown in Table 5, and these were integrated by the stepwise graphical method to determine the N.T.U. needed for the required gas absorption.

Table 4 and Table 5 show that at a tie-line slope of 0.01 (i.e. equal to the negative of the operating line slope) the values of $s_Y = s_X$, and $t_Y = t_X$ and the N.T.U. are 14.550 in both cases. For the case in which the gas-film is more important and the tie-line slope is less than 0.01 the N.T.U. becomes very large. For the case in which the liquid-film is more important and the tie-line slope is greater than 0.01 the N.T.U. also becomes very large. This result occurs because the s_X and t_X become nearly the same and the slopes s_Y and t_Y become nearly the same. The results obtained in Table 4 and Table 5 are shown in Figure 8. The extremely large N.T.U. values above 14.550 are incorrect and should not be used. The point Q in Figure 8 can be called the critical point, because at this point the procedure must be changed.

Table 4. Calculation of N.T.U. for Various Tie-Line Slopes. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. The operating line slope is 0.01. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 and leaves at 1.20 pounds of water per pound of H_2SO_4. Isothermal operation.

Gas film more important: (Y-axis direction driving forces)

Operating line slope $\frac{L'}{V'} = \frac{0.010 - 0.003}{1.20 - 0.50} = 0.010$

Tie-line slope $-\frac{k_X}{k_Y}$	s_Y	t_Y	N.T.U. graphical stepwise method	N.T.U. graphical integration method
0.0002	-0.00010	-0.00030		
0.0005	-0.00024	-0.00077		
0.001	-0.00048	-0.00158	70.000	
0.002	-0.00091	-0.00333	42.490	
0.005	-0.00200	-0.01000	21.143	
*0.01	-0.00333	-0.03000	**14.550	***14.626
0.02	-0.00500	+ ∞	10.476	
0.05	-0.00714	+0.05000	8.556	
0.1	-0.00833	+0.03750	7.838	
0.2	-0.00909	+0.03333	7.470	
0.5	-0.00962	+0.03125	7.222	
1.0	-0.00981	+0.03061	7.250	7.125
5.0	-0.00996	+0.03012		
10.0	-0.00998	+0.03006		
20.0	-0.00999	+0.03003	7.120	
50.0	-0.00999	+0.03001		
∞	-0.01000	+0.03000	7.105	

*At this point, $-\frac{k_X}{k_Y} = \frac{L'}{V'} = 0.01$.

**At this point, the N.T.U. by White's Method (10) agrees within 2.37%.

***Table 6 shows this graphical integration.

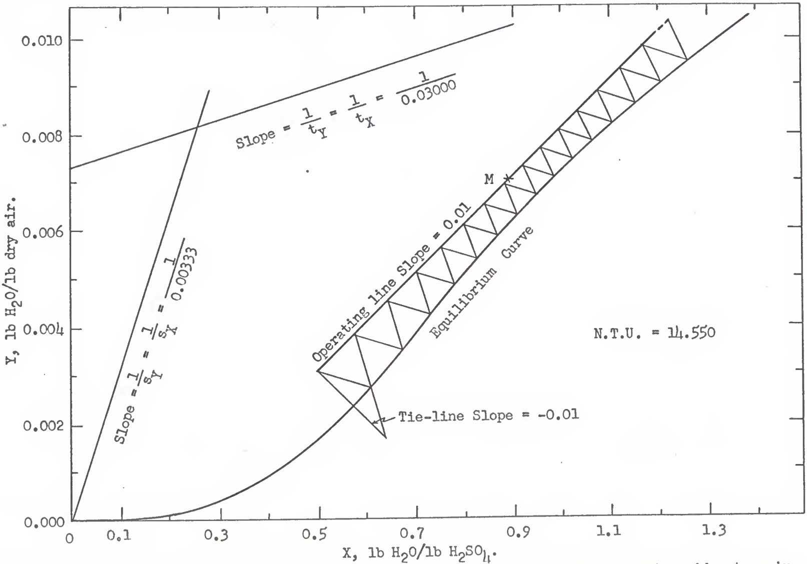

Fig. 7. Graphical stepwise construction for gas absorption. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. The operating line slope is 0.01 and the tie-line slope is -0.01. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 and leaves at 1.20 pounds of water per pound of H_2SO_4. Isothermal operation.

Table 5. Calculation of N.T.U. for Various Tie-Line Slopes. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. The operating line slope is 0.01. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 and leaves at 1.20 pounds of water per pound of H_2SO_4. Isothermal operation.

Liquid film more important:(X-axis direction driving forces)

Operating line slope $\frac{L'}{V'} = \frac{0.010 - 0.003}{1.20 - 0.50} = 0.010$

Tie-line slope $-\frac{k_X}{k_Y}$	s_X	t_X	N.T.U. graphical stepwise method
0.0000	+0.00333	-0.01000	7.279
0.0001	+0.00327	-0.01020	
0.0002	+0.00320	-0.01040	7.432
0.0005	+0.00300	-0.01100	7.843
0.001	+0.00267	-0.01200	8.390
0.002	+0.00200	-0.01400	8.788
0.005	0.00000	-0.02000	11.214
*0.01	-0.00333	-0.03000	14.550
0.02	-0.01000	-0.05000	20.328
0.05	-0.03000	-0.11000	38.000
0.1	-0.06333	-0.21000	

*At this point, $-\frac{k_X}{k_Y} = \frac{L'}{V'} = 0.01$,

and $s_X = s_Y = -0.00333$, $t_X = t_Y = -0.03000$.

Table 6. Graphical Integration of Gas Absorption. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. The operating line slope is 0.01 and the tie-line slope is -0.01. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 and leaves at 1.20 pounds of water per pound of H_2SO_4. Isothermal operation.

Gas film more important:

Operating line slope $\frac{L'}{V'} = 0.010$

$$N_{tY} = \int_{0.003}^{0.010} \frac{dY}{(1 - Y)(Y - Y_i)} = 14.626$$

Y	$1 - Y$	Y	$Y - Y_i$	$\frac{1}{(1-Y)(Y-Y_i)}$	ΔY	Ave. ordinate	Area
0.0030	0.9970	0.00225	0.00075	1333			
					0.0005	1352.5	0.6763
0.0035	0.9965	0.00277	0.00073	1372			
					0.0005	1401.0	0.7005
0.0040	0.9960	0.00330	0.00070	1430			
					0.0005	1527.5	0.7636
0.0045	0.9955	0.00388	0.00062	1625			
					0.0005	1744.5	0.8723
0.0050	0.9950	0.00446	0.00054	1864			
					0.0005	1939.5	0.9698
0.0055	0.9945	0.00500	0.00050	2015			
					0.0005	2127.5	1.0638
0.0060	0.9940	0.00555	0.00045	2240			
					0.0005	2379.5	1.1898
0.0065	0.9935	0.00610	0.00040	2515			
					0.0005	2519.5	1.2598
0.0070	0.9930	0.00660	0.00040	2520			
					0.0005	2586.0	1.2930
0.0075	0.9925	0.00712	0.00038	2652			
					0.0005	2586.0	1.2930
0.0080	0.9920	0.00760	0.00040	2520			
					0.0005	2435.0	1.2185
0.0085	0.9915	0.00810	0.00040	2520			
					0.0005	2435.0	1.2185
0.0090	0.9910	0.00857	0.00043	2350			
					0.0005	2184.5	1.0928
0.0095	0.9905	0.00900	0.00050	2019			
					0.0005	1944.5	0.9723
0.0100	0.9900	0.00946	0.00054	1870			
						Total =	14.6257

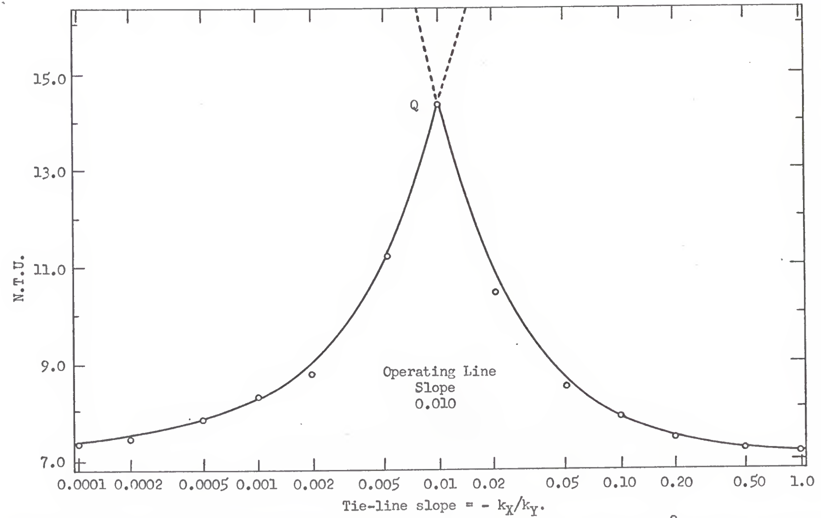

Fig. 8. N.T.U. vs. tie-line slope. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. Isothermal operation. The operating line slope is 0.01. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 and leaves at 1.20 pounds of water per pound of H_2SO_4.

Table 7 shows what happens when the operating line slope is changed to 0.00934, with the gas entering at 0.010 and leaving at 0.003 pounds of water per pound of dry air as before, and the liquid entering at 0.05 pounds of water per pound of H_2SO_4 and leaving at 1.25.

Table 8 shows what happens when the operating line slope is changed to 0.0117. Gas enters and leaves at the same compositions as before. Liquid enters at 0.5 and leaves at 1.10 pounds of water per pound of H_2SO_4.

Table 9 shows what happens when the operating line slope is increased to 0.014. Gas enters and leaves at the same compositions as before. Liquid enters at 0.5 and leaves at 1.0 pound of water per pound of H_2SO_4.

Note that in all four cases the terminal compositions at the top of the tower are the same, and those at the bottom are different.

The results obtained in Tables 4, 5, 7, 8, and 9 were combined in Table 10 and plotted in Figure 9 and 10. The N.T.U. was the largest in each case when the tie-line slope was equal to the operating line slope and of opposite sign. Figure 9 shows the various operating line slopes. Figure 10 shows the N.T.U. values vs. the tie-line slopes. The critical point corresponds to the largest N.T.U. values. Figure 9 and 10 shows that the distance between the operating line and the equilibrium curve is an indication of the driving forces. The greater the slope of the operating line, the greater will be the driving force, and the smaller the N.T.U.

Table 7. Calculation of N.T.U. for Various Tie-Line Slopes. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. The operating line slope is 0.00934. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 and leaves at 1.25 pounds of water per pound of H_2SO_4. Isothermal operation.

Operating line slope $\frac{L'}{V'} = \frac{0.010 - 0.003}{1.25 - 0.50} = 0.00934$

Liquid film more important:

Tie-line slope $-\frac{k_X}{k_Y}$	s_X	t_X	N.T.U. graphical stepwise method
0.0000	+0.00311	−0.00934	9.878
0.0001	+0.00305	−0.00954	9.834
0.0002	+0.00298	−0.00974	
0.0005	+0.00278	−0.01034	10.253
0.001	+0.00245	−0.01134	10.654
0.002	+0.00178	−0.01334	11.765
0.005	−0.00022	−0.01934	14.645
0.00934	−0.00311	−0.02802	19.143
Gas film more important:	s_Y	t_Y	
0.00934	−0.00311	−0.02814	19.143
0.01	−0.00325	−0.03244	
0.02	−0.00482	+0.39840	13.778
0.05	−0.00678	+0.04424	11.190
0.1	−0.00784	+0.03425	10.067
0.2	−0.00852	+0.03079	9.388
0.5	−0.00897	+0.02900	
1.0	−0.00914	+0.02841	9.296
10.0	−0.00928	+0.02799	
50.0	−0.00930	+0.02790	
∞	−0.00934	+0.02802	9.222

Table 8. Calculation of N.T.U. for Various Tie-Line Slopes. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. The operating line slope is 0.0117. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.5 and leaves at 1.10 pounds of water per pound of H_2SO_4. Isothermal operation.

Operating line slope $\frac{L'}{V'} = \frac{0.010 - 0.003}{1.10 - 0.50} = 0.0117$

Liquid film more important:

Tie-line slope $-\frac{k_X}{k_Y}$	s_X	t_X	N.T.U. graphical stepwise method
0.0000	+0.0039	-0.0117	3.790
0.0001	+0.0038	-0.0119	3.814
0.0002	+0.0038	-0.0121	
0.0005	+0.0036	-0.0127	4.037
0.001	+0.0032	-0.0137	4.278
0.002	+0.0026	-0.0157	4.598
0.005	+0.0006	-0.0217	5.803
0.01	-0.0028	-0.0317	
0.0117	-0.0039	-0.0351	8.175
Gas film more important:	s_Y	t_Y	
0.0117	-0.0039	-0.0351	8.175
0.02	-0.0054	-0.2065	6.489
0.05	-0.0080	+0.0660	5.130
0.1	-0.0095	+0.0458	4.738
0.2	-0.0105	+0.0398	4.432
0.5	-0.0112	+0.0368	
1.0	-0.0114	+0.0359	4.230
10.0	-0.0117	+0.0352	
50.0	-0.0117	+0.0351	
∞	-0.0117	+0.0351	4.321

Table 9. Calculation of N.T.U. for Various Tie-Line Slopes. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. The operating line slope is 0.014. Gas enters at 0.010 and leaves at 0.003 pounds of water per pound of dry air. Liquid enters at 0.50 and leaves at 1.0 pound of water per pound of H_2SO_4. Isothermal operation.

Operating line slope $\frac{L'}{V'} = \frac{0.010 - 0.003}{1.0 - 0.50} = 0.014$

Tie-line slope $-\frac{k_X}{k_Y}$	s_X	t_X	N.T.U. graphical stepwise method
Liquid film more important:			
0.0000	+0.0047	-0.0140	2.430
0.0001	+0.0046	-0.0142	
0.0002	+0.0045	-0.0144	
0.0005	+0.0043	-0.0150	
0.001	+0.0040	-0.0160	
0.002	+0.0033	-0.0180	3.129
0.005	+0.0013	-0.0240	3.526
0.01	-0.0020	-0.0340	
0.014	-0.0047	-0.0420	5.850
Gas film more important:	s_Y	t_Y	
0.014	-0.0047	-0.0420	5.850
0.02	-0.0058	-0.1050	
0.05	-0.0099	+0.0954	4.120
0.1	-0.0109	+0.0583	3.574
0.2	-0.0123	+0.0488	
0.5	-0.0133	+0.0445	
1.0	-0.0136	+0.0432	3.267
10.0	-0.0140	+0.0421	3.260
50.0	-0.0140	+0.0420	
∞	-0.0140	+0.0419	3.400

Table 10. N.T.U. vs. Tie-Line slope for Various Operating Line Slopes. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. Isothermal operation.

Operating line					Tie-line slope	N.T.U. graphical stepwise method
Slope $\frac{L'}{V'}$	X-axis liquid enter $\frac{\text{lb } H_2O}{\text{"} H_2SO_4}$	Y-axis gas leave $\frac{\text{lb } H_2O}{\text{lb air}}$	X-axis liquid leave $\frac{\text{lb } H_2O}{\text{"} H_2SO_4}$	Y-axis gas enter $\frac{\text{lb } H_2O}{\text{lb air}}$	$-\frac{k_X}{k_Y}$	
0.00934	0.50	0.003	1.25	0.010	0.0000	9.878
					0.0001	9.834
					0.0005	10.253
					0.001	10.654
					0.002	11.765
					0.005	14.645
					0.00934	19.143
					0.02	13.778
					0.05	11.190
					0.1	10.067
					0.2	9.388
					1.0	9.296
					∞	9.222
0.01	0.50	0.003	1.20	0.010	0.0000	7.279
					0.0002	7.432
					0.0005	7.843
					0.001	8.390
					0.002	8.788
					0.005	11.214
					0.01	14.550
					0.02	10.476
					0.05	8.556
					0.1	7.838
					0.2	7.470
					1.0	7.250
					20.0	7.120
					∞	7.105
0.0117	0.50	0.003	1.10	0.010	0.0000	3.790
					0.0001	3.814
					0.0005	4.037
					0.001	4.278
					0.002	4.598
					0.005	5.803
					0.0117	8.175
					0.02	6.489
					0.05	5.130
					0.1	4.738
					0.2	4.432
					1.0	4.230
					∞	4.321

Table 10. (concl.)

0.014	0.50	0.003	1.00	0.010	0.0000	2.430
					0.002	3.129
					0.005	3.526
					0.014	5.850
					0.05	4.120
					0.1	3.574
					1.0	3.267
					10.0	3.260
					∞	3.400

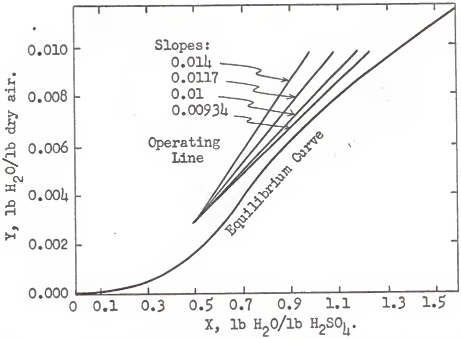

Fig. 9. Various operating line slopes in the system of sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. Isothermal operation.

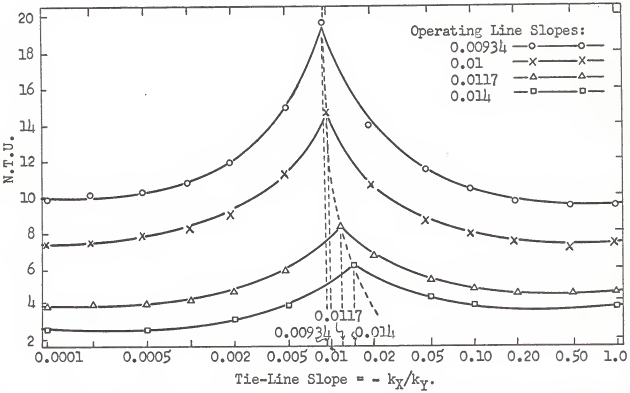

Fig. 10. N.T.U. vs. tie-line slope for various operating line slopes. The system is sulfuric acid-water-air at 25°C. and 1.0 atmosphere pressure. Isothermal operation.

The Half Way Points. A possible method for estimating the tie-line slope, $\frac{k_X}{k_Y}$, depends upon the determination, at some intermediate point in the tower, of corresponding values of X and Y. One practical experimental arrangement is to divide the tower into top and bottom sections of equal packed height, providing in effect two identical superimposed towers in series. Obviously, the point corresponding to the intermediate analysis will fall on the operating line, and there will be an equal number of transfer units in the two sections. Using trial values of the ratio, $\frac{k_X}{k_Y}$, the data can be analysed by the graphical step construction method, or by the conventional graphical integration. The proper value of the ratio is the one yielding equal numbers of transfer units in the two sections.

For the case in which the operating line slope is 0.01 and the tie-line slope is -0.01 in the system using sulfuric acid-water-air, the N.T.U. is 14.550 as shown in Figures 7 and 10 or Tables 4, 5, and 10. The half number of N.T.U. is 7.275; this point designated as M, is shown in Figure 7. The point M will fall on the operating line at the composition: $X = 0.895 \frac{\text{lb } H_2O}{\text{lb } H_2SO_4}$, $Y = 0.00695 \frac{\text{lb } H_2O}{\text{lb dry air}}$. For the same operating line slope and different tie-line slopes, -0 and -∞, the calculated half way points indicate differences of only 6.3 % of the entire change in the liquid composition in the tower. These results are shown in Table 11.

Table 11. The Half Way Point of the Same Operating Line for Three Different Tie-Line Slopes. The system is sulfuric acid-water-air st 25°C. and 1.0 atmosphere pressure.

Operating Line					Tie-Line	Half Way Point	
Slope $\frac{L'}{V'}$	X-axis liquid enter $\frac{lb\ H_2O}{''H_2SO_4}$	Y-axis gas leave $\frac{lb\ H_2O}{lb\ air}$	X-axis liquid leave $\frac{lb\ H_2O}{''H_2SO_4}$	Y-axis gas enter $\frac{lb\ H_2O}{lb\ air}$	Slope $-\frac{k_X}{k_Y}$	X $\frac{lb\ H_2O}{''H_2SO_4}$	Y $\frac{lb\ H_2O}{''H_2SO_4}$
0.01	0.50	0.003	1.20	0.010	0.00	0.861	0.00662
					0.01	0.895	0.00695
					∞	0.905	0.00705

An Equilibrium Curve That Is Concave Upward
Water-Ammonia-Air System

Calculation of Number of Transfer Units. The equilibrium curve of this system is concave upward. The equilibrium in this system was calculated from data based upon McCabe and Smith (16), as shown in Table 12. This calculation was based on the assumptions of isothermal operation at 30°C. and that the mass transfer takes place without chemical reaction.

Since the enthalpy of solution of ammonia in water is considerable it is desireable to investigate adiabatic conditions. The equilibrium data for adiabatic conditions together with the temperatures reached are given in Table 13. This calculation was based on the formation of equilibrium solutions of ammonia in water starting with pure water and ammonia gas both at 30°C. The necessary data were taken from Hougen et. al. (15). Figure 11 shows the equilibrium curve for this system under isothermal and adiabatic conditions.

Table 14 shows the results of calculations for the N.T.U.'s when the operating line slope is 4.0625. Gas enters at 0.90 and leaves at 0.25 pounds of ammonia per pound of dry air. Liquid enters at 0.07 and leaves at 0.23 pounds of ammonia per pound of water. Figure 12 illustrates the graphical stepwise procedure for the case where slopes of both operating line and tie-lines are 3.610. Table 15 shows the results for this operating line and various tie-line slopes. Gas enters and leaves the same as before. Liquid enters at 0.09 and leaves

Table 12. Calculation of X vs. Y for the System of Water-Ammonia-Air at 30°C. and 1.0 Atmosphere Pressure. Isothermal operation.

P'_A, mm. Hg	C'_A, $\frac{\text{lb NH}_3}{\text{100 lb H}_2\text{O}}$	$X=\frac{C'_A/17.03}{C'_A/17.03+5.55}$	$Y=\frac{P'_A}{760}$
10.0	1.0	0.0105	0.0132
19.0	2.0	0.0208	0.0250
29.0	3.0	0.0308	0.0382
40.0	4.0	0.0407	0.0526
50.0	5.0	0.0505	0.0658
62.0	6.0	0.0598	0.0816
86.0	8.0	0.0782	0.1131
104.0	10.0	0.0959	0.1369
175.0	15.0	0.1371	0.2303
250.0	20.0	0.1750	0.3290
360.0	25.0	0.2095	0.4740
450.0	30.0	0.2410	0.5925
560.0	35.0	0.2700	0.7360
710.0	40.0	0.2978	0.9340

Table 13. Results of the Calculation of the Equilibrium Curve in the System Water-Ammonia-Air Under Adiabatic Conditions. Starting With Pure Water and Ammonia Gas Both at 30°C.

Temperature °C.	X $\frac{\text{lb } NH_3}{\text{lb } H_2O}$	Y $\frac{\text{lb } NH_3}{\text{lb dry air}}$
30	0.020	0.030
40	0.048	0.089
50	0.065	0.195
55	0.075	0.275
60	0.088	0.404
65	0.100	0.580
70	0.110	0.752
75	0.111	0.950

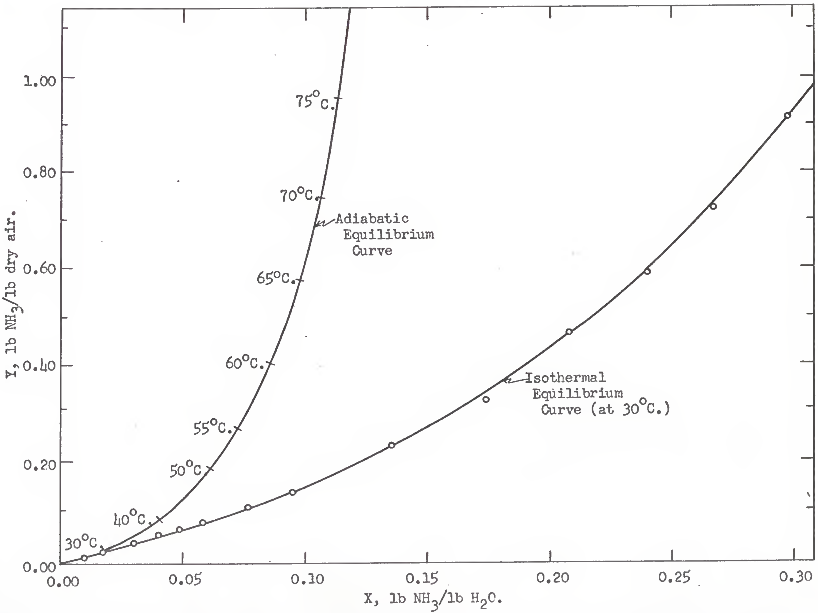

Fig. 11. The adiabatic and isothermal equilibrium curves of water-ammonia-air system.

Table 14. Calculation of N.T.U. for Various Tie-Line Slopes. The system is Water-ammonia-air at 30°C. and 1.0 atmosphere pressure. The operating line slope is 4.0625. Gas enters at 0.90 and leaves at 0.25 pounds of ammonia per pound of dry air. Liquid enters at 0.07 and leaves at 0.23 pounds of ammonia per pound of water. Isothermal operation.

Operating line slope $\frac{L'}{V'} = \frac{0.90 - 0.25}{0.23 - 0.07} = 4.0625$

Liquid film more important:

Tie-line slope $-\frac{k_X}{k_Y}$	s_X	t_X	N.T.U. graphical stepwise method
0.00	+1.3542	-4.0625	2.002
0.01	+1.3475	-4.0825	2.002
0.02	+1.3408	-4.1025	
0.05	+1.3208	-4.1625	
0.10	+1.2875	-4.2625	2.080
0.20	+1.2208	-4.4625	2.126
0.50	+1.0208	-5.0625	2.272
1.0	+0.6875	-6.0625	2.545
2.0	+0.0208	-8.0625	3.175
4.0625	-1.3542	-12.1875	4.240
Gas film more important:	s_Y	t_Y	
4.0625	-1.3542	-12.1875	4.240
5.0	-1.5475	-19.5059	3.928
10.0	-2.2411	+64.9351	3.120
20.0	-2.8885	+20.5198	2.793
50.0	-3.4941	+14.5490	2.558
100.0	-3.7566	+13.2626	2.473
200.0	-3.9032	+12.7011	2.465
500.0	-4.0290	+12.2850	
∞	-4.0625	+12.1875	2.408

Table 15. Calculation of N.T.U. for Various Tie-Line Slopes. The system is Water-ammonia-air at 30°C. and 1.0 atmosphere pressure. The operating line slope is 3.610. Gas enters at 0.90 and leaves at 0.25 pounds of ammonia per pound of dry air. Liquid enters at 0.09 and leaves at 0.27 pounds of ammonia per pound of water. Isothermal operation.

Operating line slope $\frac{L'}{V'} = \frac{0.90 - 0.25}{0.27 - 0.09} = 3.610$

Liquid film more important:

Tie-line slope $-\frac{k_X}{k_Y}$	s_X	t_X	N.T.U. graphical stepwise method
0.00	+1.2033	-3.6100	3.914
0.01	+1.1967	-3.6300	
0.02	+1.1900	-3.6500	
0.05	+1.1700	-3.7100	3.879
0.1	+1.1367	-3.8100	3.870
0.2	+1.0700	-4.0100	4.114
0.5	+0.8700	-4.6100	4.410
1.0	+0.5367	-5.6100	4.851
2.0	-0.1300	-7.6100	5.446
*3.61	-1.2033	-10.8300	7.325
Gas film more important:	s_Y	t_Y	
*3.61	-1.2033	-10.8300	7.325
5.0	-1.4742	-24.3902	6.429
10.0	-2.0964	+38.9610	5.000
20.0	-2.6525	+16.9492	4.346
50.0	-3.1546	+12.8205	4.000
100.0	-3.3670	+11.7510	3.804
200.0	-3.4843	+11.2360	
500.0	-3.5842	+10.9091	
∞	-3.6100	+10.8300	3.657

*At this condition $-\frac{k_X}{k_Y} = \frac{L'}{V'} = 3.610$ and $s_X = s_Y = -1.2033$, and $t_X = t_Y = -10.8300$. The graphical stepwise construction shown in Figure 12.

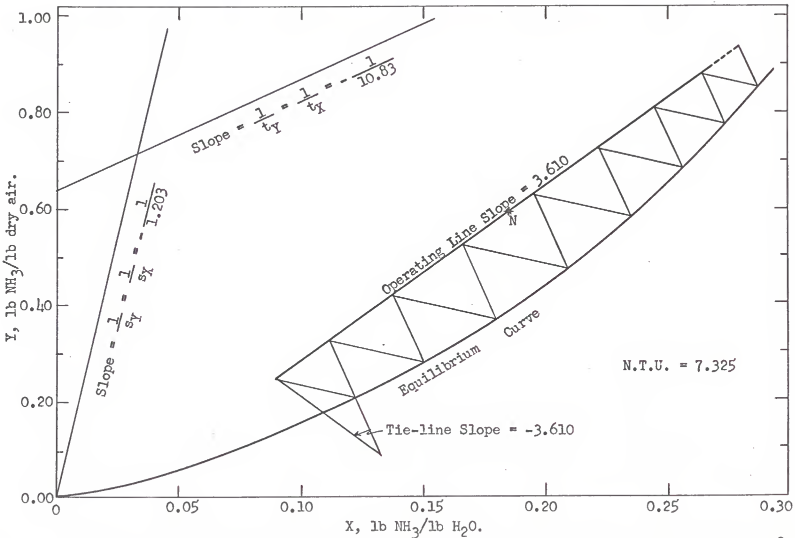

Fig. 12. Graphical stepwise construction of gas absorption. The system is water-ammonia-air at 30°C. and 1.0 atmosphere pressure. The operating line slope is 3.610. Gas enters at 0.90 and leaves at 0.25 pounds of ammonia per pound of dry air. Liquid enters at 0.09 and leaves at 0.27 pounds of ammonia per pound of water. Isothermal operation.

at 0.27 pounds of ammonia per pound of water. Table 16 shows what happens when the operating line slope decreases further to 3.250. Gas enters and leaves the same as before. Liquid enters at 0.07 pounds of ammonia per pound of water and leaves at 0.27.

Note that the same terminal compositions exist at the top of the tower in the first and third cases and at the bottom of the tower in the second and third cases.

The results of Table 14, Table 15, and Table 16 are summarized in Table 17 and Figure 13, and 14. The number of transfer units was the largest when the tie-line slope was equal to the operating line slope and of opposite sign. Figure 12 shows the water-ammonia-air system for this case.

The Half Way Points. The half way points on the operating line were calculated in the same way as that outlined for the sulfuric acid-water-air system. For the conditions of operation shown in Figure 12 the half way point N corresponds to a tie-line slope of -3.610 and an operating line slope of 3.610. If the tie-line slopes are shifted from -0 to -∞ the half way point changes by an amount equivalent to 10.45% of the entire change in liquid composition in the tower. This variation is somewhat more favorable than the case investigated in Figure 7. The indicated tie-line slope will be more accurate than before. These results are shown in Table 18.

Table 16. Calculation of N.T.U. for Various Tie-Line Slopes. The system is water-ammonia-air at 30°C. and 1.0 atmosphere pressure. The operating line slope is 3.250. Gas enters at 0.90 and leaves at 0.25 pounds of ammonia per pound of dry air. Liquid enters at 0.07 and leaves at 0.27 pounds of ammonia per pound of water. Isothermal operation.

Operating line slope $\frac{L'}{V'} = \frac{0.90 - 0.25}{0.27 - 0.07} = 3.250$

Liquid film more important:

Tie-line slope $-\frac{k_X}{k_Y}$	s_X	t_X	N.T.U. graphical stepwise method
0.00	+1.0833	-3.2500	3.710
0.01	+1.0767	-3.2700	3.710
0.02	+1.0700	-3.2900	
0.05	+1.0500	-3.3500	
0.1	+1.0167	-3.4500	3.652
0.2	+0.9500	-3.6500	3.870
0.5	+0.7500	-4.2500	4.125
1.0	+0.4167	-5.2500	4.606
2.0	-0.2730	-7.2500	5.683
3.250	-1.0833	-9.7500	6.674
Gas film more important:	s_Y	t_Y	
3.250	-1.0833	-9.7500	6.674
5.0	-1.4130	-32.5027	5.438
10.0	-1.9697	+27.8552	4.275
20.0	-2.4528	+14.4439	3.724
50.0	-2.8760	+11.2066	3.407
100.0	-3.0516	+10.4275	3.293
200.0	-3.1476	+10.0773	
500.0	-3.2289	+9.8135	
∞	-3.2500	+9.7500	3.164

Table 17. N.T.U. vs. Tie-Line Slope for Various Operating Line Slopes. The system is water-ammonia-air at 30°C. and 1.0 atmosphere pressure. Isothermal operation.

	Operating line					
Slope $\frac{L'}{V'}$	X-axis liquid enter $\frac{lb\ NH_3}{lb\ H_2O}$	Y-axis gas leave $\frac{lb\ NH_3}{lb\ air}$	X-axis liquid leave $\frac{lb\ NH_3}{lb\ H_2O}$	Y-axis gas enter $\frac{lb\ NH_3}{lb\ air}$	Tie-line slope $-\frac{k_X}{k_Y}$	N.T.U. graphical stepwise method
4.0625	0.07	0.25	0.23	0.90	0.00	2.002
					0.10	2.080
					0.20	2.126
					0.50	2.272
					1.00	2.545
					2.00	3.175
					4.0625	4.240
					5.00	3.928
					10.00	3.120
					20.00	2.793
					50.00	2.558
					100.00	2.473
					∞	2.408
3.610	0.09	0.25	0.27	0.90	0.00	3.914
					0.20	4.114
					0.50	4.410
					1.00	4.851
					2.00	5.446
					3.610	7.325
					5.00	6.429
					10.00	5.000
					20.00	4.346
					50.00	4.000
					100.00	3.804
					∞	3.657
3.250	0.07	0.25	0.27	0.90	0.00	3.710
					0.10	3.652
					0.20	3.870
					0.50	4.125
					1.00	4.606
					2.00	5.683
					3.250	6.674
					5.00	5.438
					10.00	4.275
					20.00	3.724
					50.00	3.407
					100.00	3.293
					∞	3.164

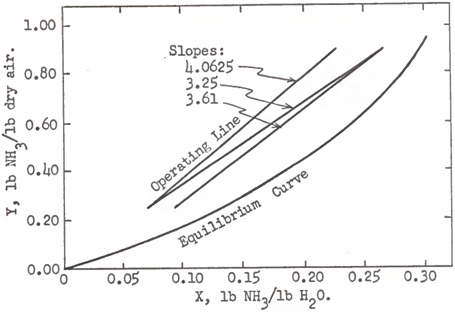

Fig. 13. Various operating line slopes in the system of water-ammonia-air at 30°C. and 1.0 atmosphere pressure. Isothermal operation.

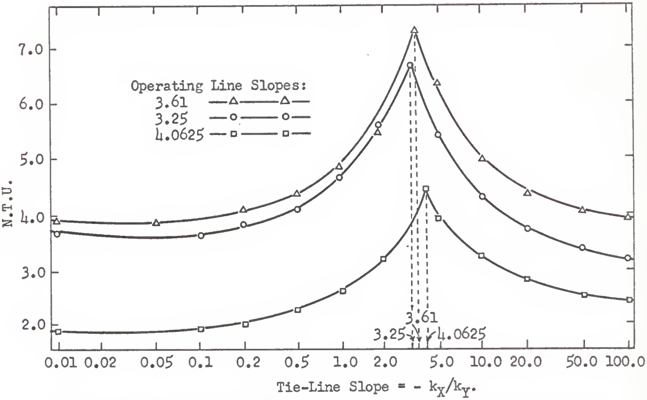

Fig. 14. N.T.U. vs. tie-line slope for various operating-line slopes. The system is water-ammonia-air at 30°C. and 1.0 atmosphere pressure. Isothermal operation.

Table 18. The Half Way Point of the Same Operating Line for Three Different Tie-Line Slopes. The system is water-ammonia-air at 30°C. and 1.0 atmosphere pressure.

	Operating Line				Tie-Line	Half Way Point	
Slope $\frac{L'}{V'}$	X-axis liquid enter $\frac{\text{lb } NH_3}{\text{lb } H_2O}$	Y-axis gas leave $\frac{\text{lb } NH_3}{\text{lb air}}$	X-axis liquid leave $\frac{\text{lb } NH_3}{\text{lb } H_2O}$	Y-axis gas enter $\frac{\text{lb } NH_3}{\text{lb air}}$	Slope $-\frac{k_X}{k_Y}$	X $\frac{\text{lb } NH_3}{\text{lb } H_2O}$	Y $\frac{\text{lb } NH_3}{\text{lb air}}$
3.610	0.09	0.25	0.27	0.90	0.00	0.200	0.650
					3.610	0.182	0.600
					∞	0.177	0.561

SUMMARY

In a gas absorption process the operating line lies above the equilibrium curve. In order to make the operating line straight all the graphs should use mole or weight ratios rather than mole or weight fractions. Evaluation of the number of transfer units by White's method gave very satisfactory results compared with the formal graphical integration method. White's method is very quick so that a number of integrations can be made in a short time. In this way much insight can be gained into the operation of gas absorption towers. It is applicable to all tie-line slopes between zero and infinity. There is a critical tie-line slope equal to the operating line slope and of opposite sign. This critical tie-line slope divides the White method into two kinds of steps which are related to the cases in which the gas-film resistance or the liquid-film resistance is most important. At the critical tie-line slope the number of transfer units reaches a maximum and the absorption tower height a maximum.

Two systems were used in the exploratory calculations. The sulfuric acid-water-air system has an S-shaped equilibrium curve while the water-ammonia-air system has an equilibrium curve that is concave upward. These are for isothermal operation. The heat of solution for both systems is considerable. Thus, adiabatic operation results in both systems having equilibrium curves that are concave upward.

In those cases where the equilibrium curve exhibits considerable curvature there is a method of analysing experimental data to determine the approximate tie-line slope. This means that the relative resistances on the liquid-film side and on the gas-film side of the gas-liquid interface can be found. Such information would be quite useful in the design of gas absorption towers.

In this report the scheme proposed is to divide the gas absorption system into two identical towers in series-counterflow. The gas and liquid compositions at the half way point would be determined as well as those at the top and bottom of the system. A tie-line slope would be assumed and the integration for the number of transfer units would be determined by White's method. This would also give the half way point on the operating line. If the half way points did not agree a new tie-line slope would be tried. The correct tie-line slope would make the half way points identical.

For the conditions tried, the variation in the tie-line slope from zero to infinity changed the half way point on the operating line 6 to 10% of its length. The sulfuric acid-water-air system yielded a variation of 6.3% and the water-ammonia-air system a variation of 10.45%. However, it is probable that these figures depend upon the terminal conditions. The true half way point must lie in this range. This gives an idea of the probable accuracy in estimating the true tie-line slope.

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LITERATURE CITED

1. Colburn, A. P., Trans. A.I.Ch.E. Journal, 35, 211 (1939).
2. Ibid., Ind. Eng. Chem., 33, 459 (1941).
3. Onda, K., and Sada, E., Kagaku Kogaku, Vol. 21, No. 8, 28 (1957).
4. Ramm, W. M., "Absorptionsprozesse in der Chemischen Tecknik", 2 ed., p. 91, Verlag Tecknik, Berlin (1953).
5. Scheibel, E. G., and Othmer, D. F., Ind. Eng. Chem., 34, 1200 (1942).
6. Ibid., Trans. A.I.Ch.E. Journal, 38, 339 (1942).
7. Wiegand, J. H., Trans. A.I.Ch.E. Journal, 36, 679 (1940).
8. White, G. E., Chem. Eng. Progr., 46, 363 (1950).
9. Baker, E. M., Ind. Eng. Chem., 27, 977 (1935).
10. White, G. E., Trans. A.I.Ch.E. Journal, 36, 359 (1940).
11. Ogawa, S., Kagaku Kikai, Vol. 9, No. 10, 18 (1945).
12. Whitman, W. G., Chem. Met. Eng., 29, 147 (1923).
13. Greenewalt, C. H., Ind. Eng. Chem., 17, 522 (1925).
14. Landolt-Börnstein, "Physikalish-Chemisch Tabellen", 5 Aulfe II, p. 1394, Edited by W.A.Roth & K.Scheel and numerous other collaborators, Julius Springer, Berlin (1923).
15. Hougen, O. A., Watson, K. M., and Ragatz, R. A., "Chemical Process Principles", Part I, Material and Energy Balances, 2 ed., p. 297, 301, 320, John Wiley & Sons, New York (1954).
16. McCabe, W. L., and Smith, J. C., "Unit Operations of Chemical Engineering", p. 638, McGraw-Hill, New York (1956).

NOMENCLATURE

k = film coefficient of mass transfer, lb moles/(hr)(sq.ft).

L' = lb inert liquid per unit time, lb/hr.

N.T.U. = number of transfer units.

s = slope of lower trace line in graphical construction of transfer unit.

t = slope of upper trace line in graphical construction of transfer unit.

V' = lb inert gas per unit time, lb/hr.

X = lb ratio of solute to lb inert liquid.

X_i = lb ratio of solute at gas-liquid interface.

X^* = equilibrium value of X corresponding Y.

Y = lb ratio of solute to lb inert gas.

Y_i = lb ratio of solute at gas-liquid interface.

Y^* = equilibrium value of Y corresponding to X.

Subscripts:

o = overall.

X = liquid, or liquid-film or based on X units.

Y = gas, or gas-film, or based on Y units.

THE EFFECT OF CURVATURE OF THE EQUILIBRIUM LINE ON THE FILM RESISTANCES IN GAS ABSORPTION

by

CHAO-JEN KAO

B. S., Taiwan Provincial Cheng-Kung University, 1951

AN ABSTRACT OF A MASTER'S REPORT

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1964

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